CHARACTERISTICS AND APPLICATIONS OF COUPLED MICROSTRIP LINES.

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Abstract

A general formalism extending conventional Kirchhoff's line theory appears to be a convenient tool for defining the main characteries of coupled lines and for studying various applications.

Scattering Matrix of Coupled Line Section.

A coupled line section is a special case of a set of parallel coupled lines (1). Furthermore the medium will be inhomogeneous for microstrip devices.

The methodology for obtaining the scattering matrix describing a coupled line section is as follows. (Fig.1)

- The data are width W_i of each line, spacing s_i between lines, line length ℓ, substrate heigth h, substrate permittivity ε_r.

- The first step is to determine the matrices $[E]$ and $[M]$ of electric and magnetic influence cœfficients, per unit length of coupled section. One of the most powerful methods, which has been shown to be a convenient one whatever the cross-sectional geometry, appears to be an overrelaxation method (2).

Two matrices $[E_o]$ and $[E]$ are calculated, the first one as if the medium were a vacuum, the second one with the inhomogeneous dielectric $[M]$ is obtained from $[E]$ and $[E_o]$ (1).

- The influence cœfficients lead to a set of coupled propagation equations. These equations may be reduced to a single matrix equation.

A first intermediate matrix $[T]$ is there introduced, which gives a set of uncoupled propagation equations corresponding to pure waves propagatiing with separate velocities v^i ($i = 1, 2, \dots N$) if there are N coupled lines. These velocities define a diagonal matrix $[B]$ of elements $\beta^i = \omega/v^i$ (ω : radian frequency). The influence of the differences between these velocities has already been indicated (3) and will be emphasized in the applications. T is the matrix which diagonalizes $[E]$ $[M]$ and the $[v^i]^{-1/2}$ are the eigenvalues of $[E]$ $[M]$.

A second intermediate matrix $[Z_c]$ is there defined by:

$$(1) \quad [Z_c]\,[E]\,[Z_c] = [M].$$

This characteristic impedance matrix $[Z_c]$ generalizes to many dimensions, the notion of characteristic impedance.

- It has been shown that, as for a single line, the impedance matrix $[Z]$ of a coupled line section, considered as a multiport device, may be expressed using $[T]$, $[Z_c]$, $[B]$ as follows :

$$(2) \quad [Z] = \begin{bmatrix} [Z_c][T]\left[\coth[\Gamma]\ell\right][T]^{-1} - [Z_c][T]\left[\mathrm{sh}[\Gamma]\ell\right]^{-1}[T]^{-1} \\ [Z_c][T]\left[\mathrm{sh}[\Gamma]\ell\right]^{-1}[T]^{-1} - [Z_c][T]\left[\coth[\Gamma]\ell\right][T]^{-1} \end{bmatrix}$$

- If the various ports are loaded by any scalar impedance Z_o, the scattering matrix $[S]$ is given by the well-known (4) relation :

$$(S) = \left[[Z] - Z_o[1]\right]\left[[Z] + Z_o[1]\right]^{-1} \qquad (2\text{ bis})$$

$[1]$ being the unit matrix.

Attenuation and Phase Shift of a Coupled-line Section.

All the transfer properties between any pair of ports $[i]$ and $[j]$ are given by studying the elements s_{ij} of $[S]$ as function of frequency f, which is an implicit variable.

1) The attenuation is the modulus $|s_{ij}|$ of s_{ij}.

2) The phase shift is the argument of s_{ij}.

Loading Effects.

In a general manner the matching at the port i is obtained with $s_{ii} = 0$ (s_{ii} element of $[S]$). Here, as mentioned above, all the ports are loaded by an arbitrary impedance Z_o; as a results s_{ii} is an implicit function of frequency f and of the load Z_o.

Matching the input port k connected to the feeder is equivalent to solving the equation $s_{kk}\,[f, Z_o] = 0$ for f or Z_o.

Bandwidth limitations.

Assuming, at least a matching at a central frequency f_c, with a convenient load value Z_{om}, the curve $20 \ln|s_{ij}|$ as a function of frequency, will give the bandwidth, according to conventional definitions, for $Z_o = Z_{om}$.

Applications.

a - Bandwidth of a suspended symmetric microstrip coupler.

One end of the coupled section is provisionally considered as the device "input", with two input ports labeled 1, 2, one of which, labeled 1, is connected to the feeder. An imput matrix $[Z_{in}]$ relating voltages and currents at these inputs, may then be defined, with :

$$[Z_{in}] = [Z_c]\left[\left[[Z_c]+Z_o[1]\right][T]\left[e^{jB\ell}\right][T]^{-1}\right]^{-1}$$

$$(3) \quad \left[Z_o[T]+j[Z_c][T][\mathrm{tg}\,B\ell]\right] \times \left[[Z_c][T]+jZ_o[T][\mathrm{tg}\beta\ell]\right]^{-1}$$

$$\left[[Z_c] + Z_o[1]\right][T]\left[e^{jB\ell}\right][T]^{-1}$$

The matching condition at port 1 is realized if the input impedance appears to be Z_o, loading to the matching equation :

$$(4) \quad \det\left[[Z_{in}] - Z_o[D]\right] = 0$$

(det : determinant), where $[D]$ is the special matrix : $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. This equation may be solved for any frequency f, giving a complex solution $Z_{om}[f]$. With this solution, the attenuation s_{12}, defining the coupling between arms 1 and 2, has a first maximum at a frequency f_c near $\dfrac{v^e v^o}{2[v^e + v^o]\ell}$ and has zeroes for frequencies $(5)\ f' = k\left[1 + \dfrac{v^o}{v^e}\right] f_c$ ($k = 0, 1, 2 \dots$), with $Z_{om}[f] = Z_{oe}$ for $k \neq 0$, and $(6)\ f'' = k\left[1 + \dfrac{v^e}{v^o}\right] f_c$, with $Z_{om}[f] = Z_{oo}$ for $k \neq 0$.

Here v^o, v^e, Z_{oo}, Z_{oe} are respectively the usual odd and even velocities, v^i (i = 1, 2) defined above, and characteristic impedances. These latter impedances are obtained from the elements Z_{ij} of the previously defined matrix (Z_c), with $2Z_{11} = Z_{oe} + Z_{oo}$, $2Z_{12} = Z_{oe} - Z_{oo}$.

The theory is new applied to a shielded symmetric suspended microstrip coupler with w = 2.2 mm, h = 0.8mm having shield sizes : height 3.7 mm, width 32 mm. The substrate is teflon with $\varepsilon_r = 2.65$. Figure 2 shows the convenient complex load value $Z_{om}(f)$ giving the right matching over the frequency range 0 to 5GHz. Figure 3 gives, in the cases $\varepsilon_r = 1$ and $\varepsilon_r = 2.65$, the coupling cœfficient for the studied coupler of -4dB, designed to have a f_c = 1.5 GHz. It appears clearly on figure 3 that, in the homogeneous case, the differences between the eigenvelocities v^e and v^o damage the bandwith and coupling periodicity. The maxima remain near $(2n+1)f_c$, (n = 0, 1, 2 ...) but each minimum $2nf_c$ splits into the two values f' and f", whose separation increases with the difference between v^e and v^o; furthermore an unforeseen maximum appears at $2f_c$.

b - A two step phase-shifter in microstrip line.

A single meander can be realized by joining with a short circuit the two ports at one end of a two coupled line section. From the above theory it follows that the right matching load : $Z_{om} = R_{om} + jX_{om}$ is solution of (7) :

$$R_{om}^2 + (X_{om} - Z_{oe}\,\mathrm{cotg}\,\beta^e\ell)(X_{om} + Z_{oo}\,\mathrm{tg}\,\beta^o\ell) = 0.$$

The locus of Z_{om} in the complex plane is a circle centered at 0, $\dfrac{Z_{oe}\,\mathrm{cotg}\,\beta^e\ell - Z_{oo}\,\mathrm{tg}\,\beta^o\ell}{2}$ and of radius : $\dfrac{Z_{oe}\,\mathrm{cotg}\,\beta^e\ell + Z_{oo}\,\mathrm{tg}\,\beta^o\ell}{2}$. If the matching load is required to be real, then :

(8) $R_{om}^2 = Z_{oe}Z_{oo}\,\mathrm{tg}\,(\beta^o\ell)\,\mathrm{cotg}\,(\beta^e\ell)$.

This latter result has already appeared in the literature (5), based on a simplified theory, showing in the case of any inhomogeneous substrate (i.e. any microstrip device) the discrepancy of the formula :

(9) $R_{om}^2 = Z_{oe}Z_{oo}$ generally conceded. The time delay τ can be obtained directly from (9) as :

$$(10)\quad \tau = \frac{Z_{oe}/Z_{om}}{2f_c\left[\dfrac{Z_{oe}}{Z_{om}}\cos^2\left(\dfrac{\omega}{4f_c}\right) + \sin^2\left(\dfrac{\omega}{4f_c}\right)\right]}$$

(f_c : central frequency) by setting $\beta^e = \beta^o$.

A better value in quite good agreement with experimental results can be obtained by using (7) or (8). Knowing the variation of phase shift versus frequency for a single meander, the phase shift $\Delta\Phi$ between the two outputs of two different meanders connected to a single feeder has been studied. Figure 4 shows it is possible to have two steps of phase shifts by taking for the first adjacent meander $W_1/h = 0.25$, $s_1/h = 0.125$, f_{1c} and for the second : $W_2/h = 0.25$, $s_2/h = 0.5$, $f_{1c}/2$ with $\varepsilon_r = 9.7$.

The experimental results are in good agreement with the theory.

c - An optimized resonant ring filter.

Using a two coupled microstrip line section, one line is joined at its ends to a microstrip rectangular horseshoe. The resulting device is a resonant ring filter exhibiting interesting properties as a time delay equalizer. As for the coupler a central frequency f_c near $\dfrac{v^e+v^o}{8\ell}$ is chosen and the horseshoe length is approximately 3ℓ. Then, analytical calculations using the scattering matrix give as a suitable matching load:

$$(11)\quad Z_{om} = \left| Z_{11} - \frac{Z_{12}^2}{Z_{11} - Z'_c} \right|, \qquad Z'_c(W')$$

being the characteristic impedance of the microstrip horseshoe line of width W', the coupled section having two lines of width W. The computed curve of Z_{om} versus Z'_c is given in figure 5. From this curve (with some further calculations) many interesting and new results appear. The group delay curve versus frequency is a convenient and regular one only in a narrow bandwidth in Z'_c between $Z_{11} - (Z_{12}^2/Z_{11})$ and Z_{11} and it is minimum for a very special value of Z_{om} : $Z_{om}^2 = Z_{oe}Z_{oo}$ giving $Z_{om} = Z'_c$ (the matching value commonly used as a good one in the literature . Moreover $Z_{om}(Z'_c)$ varies rapidly around this "classical" matching value : $Z_{om} = (Z_{oe}Z_{oo})^{1/2}$ (Between 0 and +∞), so that a very slight mismatch from the correct value of Z_{om} has an important effect on the group delay and a drastic one on the V.S.W.R. The ignorance of this point explains Wardrop's difficulties (6) and the empirical design of Lee (7) with a tuning dielectric and adjusted dielectric slices. Finally the V.S.W.R. will be close to zero all over the frequency range if one satisfies simultaneously : $Z_{om}^2 = Z_{oe}Z_{oo}$ and $Z'^2_c = Z_{oe}Z_{oo}$.

If not, computer calculations reveal the bad V.S.W.R. observed experimentally by Wardrop. All these results emphasize the influence of the choice of W' with respect to W. Thus, it is possible to obtain a convenient design of a device having a prescribed group delay curve with a minimum V.S.W.R. .

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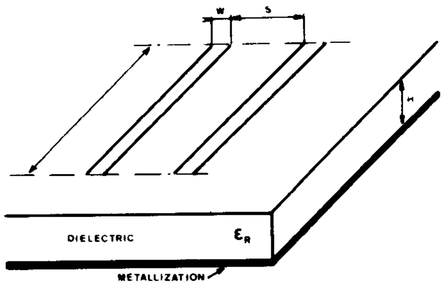

Figure 1 : Two coupled microstrip line section.

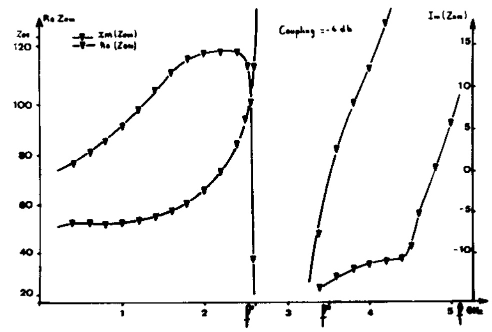

Figure 2 : Convenient load matching a coupler all over the bandwidth.

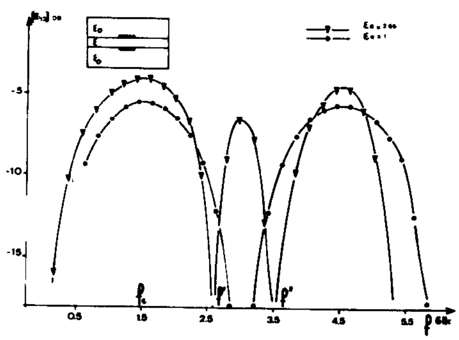

Figure 3 : Typical coupling coefficient in a vacuum and with an inhomogeneous medium.

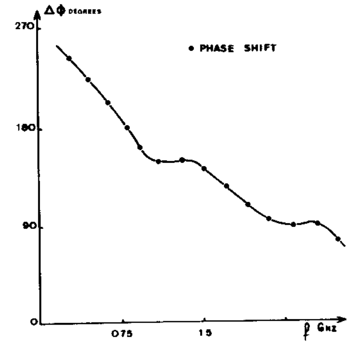

Figure 4 : Phase shift with two steps between two different microstrip meanders.

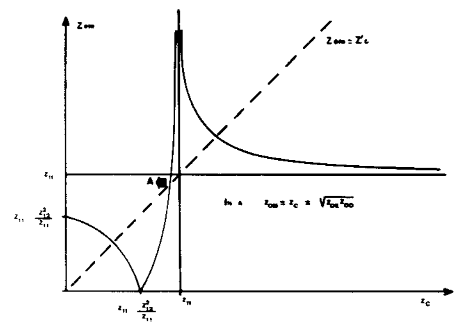

Figure 5 : Computed matching load of a resonant ring filter versus the characteristic impedance of the horseshoe.

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